Mock Exam Graph Theory

2022–2023, duration : 2 hours

- It is absolutely not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.
- Always make sure to state clearly any results from the lecture notes you are using.
- Write the answer to each question on a separate sheet, **with your name and student number on each sheet**. This is worth 10 points (out of a total of 100).

Exercise 1 (20 pts).
Prove that a planar graph in which all vertices have degree ≥ 5 satisfies $v(G) \geq 12$.

Exercise 2 (20 pts).
We consider the complete graph K_5 on five vertices, together with edge weights $w(e)$ given by the following table

$i \setminus j$	1	2	3	4	5
1					
2	1				
3	3	2			
4	7	9	10		
5	4	6	5	8	

(The table indicates $w(ij)$ for $j < i$.)

Apply Kruskal's algorithm to determine a minimum spanning tree. Make sure to clearly indicate, for each step of the algorithm, what action the algorithm takes.

Exercise 3 (30 pts)
Let G be a graph and $\overline{G}$ be its complement, i.e. $V(\overline{G}) = V(G), E(\overline{G}) = \binom{V(G)}{2} \setminus E(G)$.

a) Show that $\chi(G) \cdot \chi(\overline{G}) \geq v(G)$.

(*Hint:* How do $\omega(\overline{G})$ and $\alpha(\overline{G})$ relate to $\omega(G)$ and $\alpha(G)$?)

b) Deduce that $\chi(G) + \chi(\overline{G}) \geq 2\sqrt{v(G)}$.

Exercise 4 (20 pts).
Show that for every k there is a bipartite graph with list chromatic number $> k$.

(The end)